3rd INTERNATIONAL WORKSHOP ON NEURODYNAMICS (NDY'23)

A workshop on Neuroscience and Dynamical systems

Castro Urdiales, Cantabria, Spain
June 14-17, 2023

Scientific committee

- Valeriy Makarov
- Santiago Ibáñez
- Krasimira Tsaneva-Atanasova
- Boris Gutkin
- Stephen Coombes
- Roberto Barrio

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Contents

1 INTRODUCTION

Neuroscience is nowadays one of the most collaborative and active scientific research fields as it has been increasingly involving the participation of experts from other disciplines. In particular, computational and mathematical aspects of neuroscience are currently playing an important role both in modeling and replicating experimental findings and in explaining the underlying mechanisms of neurophysiological or cognitive processes. Differential equations are ubiquitous in the modeling of such phenomena and, consequently, nonlinear dynamics and dynamical systems techniques become fundamental sources of new mathematical and computational tools to study neuroscience models.

The aim of this Third Workshop on Neurodynamics, NDy'23 (last editions were NDy'18 and NDy'14) is to present an overview of successful achievements in this rapidly developing collaborative field by putting together different types of applications of nonlinear dynamics (geometrical tools in dynamical systems, numerical methods, computational schemes, dynamical measures,...) to different problems in neuroscience (mononeuronal dynamics, network activity, cognitive problems,...). Additional emphasis will be put on experimental findings seeking for theoretical explanations, and therefore this meeting is focussed on using mathematics as the primary tool for elucidating the fundamental mechanisms responsible for experimentally observed behavior in the applied neurosciences. Importantly, it will draw attention to those pieces of mathematical theory which are likely to be relevant to future studies of the brain. The final goal is spreading together mathematical methodology and neuroscience challenges and stimulating future cross-collaborations among participants, being Mathematical Neuroscience the generic topic for NDy'23.

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2 PROGRAMME

	Wednesday	Thursday	Friday	Saturday
9:30-10:00		J. Slawinska	C. Masoller	C. Gros
10:00-10:30		A. René	B. Pietras	L. Pérez
10:30-11:00		K. Nechyporenko	D. Noriega	G. Oleaga
11:00-11:30		*Coffee break*	*Coffee break*	*Coffee break*
11:30-12:00		A. Guillamon	Á. Byrne	M. Martin
12:00-12:30		A. Pérez-Cervera	P. Langfield	R. Vigara
12:30-13:00		P.Y. Houzelstein	J.A. Jover-Galtier	
13:00-13:30				
13:30-16:00		LUNCH	LUNCH	LUNCH
16:00-16:30	Reception	S. Coombes	V. Makarov	
16:30-17:00	A.E. Teruel	H. Schmidt	R. Nicks	
17:00-17:30	A. Windisch	A. Mayora-Cebollero	R. Allen	
17:30-18:00	A. Akhshi			
21:00		**SOCIAL DINNER**		

All the talks will be placed at the CIEM center, 4 Maria Aburto street (see the map at the end of the program).

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3 COMMITTEES

3.1 Scientific Committee

Roberto Barrio
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Computational Dynamics Group (CoDy)
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4 ABSTRACTS

Exploring Heterogeneity in Pyramidal Cell Firing Activity: A Computational Modeling Approach

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Abstract

Understanding how pyramidal cells in the sensory nervous system encode incoming information to give rise to perception and behavior remains a key challenge in systems neuroscience. This understanding is complicated, in part, by the fact that neurons ubiquitously display different patterns of spiking activity, even within the same cell type [1]. Growing evidence suggests that spiking heterogeneities can be beneficial in robust learning and behavior by increasing the capacity of information transfer [2]. Yet, the cellular and molecular mechanisms underlying this spiking heterogeneity remain poorly understood [1].

In this study, we developed a novel computational workflow in which a stochastic biophysical modeling approach and optimization technique were employed to investigate the role of different parameters of a (ghostbursting) Hodgkin-Huxley (HH) type model in defining the firing activity of pyramidal cells [3]. Our approach enabled us to match the firing activity of the HH type model with those obtained experimentally in vivo when recording membrane potential in pyramidal cells of the ELL.

Using bifurcation theory, we explained how the firing activity of pyramidal cells switches from quiescence to tonic firing and to burst firing by changes in critical parameters of the model. We also investigated how changes in multiple variables can alter the aforementioned activity in a 2-dimensional parameter space. Furthermore, we used slow-fast analysis to explain the effects of calcium on the underlying mechanism of cell dynamics undergoing burst oscillation. Finally, we investigated how the model explains the contribution of synaptic bombardments to firing, as well as how this firing is neuromodulated by serotonin through pharmacological manipulation of pyramidal cells.

References

[1] Nandi, A., Chartrand, T., Van Geit, W., Buchin, A., Yao, Z., Lee, S., Wei, Y., Kalmbach, B., Lee, B., Lein, E. & Others Single-neuron models linking

electrophysiology, morphology, and transcriptomics across cortical cell types. *Cell Reports.* **40**, 111176 (2022).

[2] Perez-Nieves, N., Leung, V., Dragotti, P. & Goodman, D. Neural heterogeneity promotes robust learning. *Nature Communications.* **12**, 5791 (2021).

[3] Doiron, B., Laing, C., Longtin, A. & Maler, L. Ghostbursting: a novel neuronal burst mechanism. *Journal Of Computational Neuroscience.* **12** pp. 5-25 (2002).

Neural oscillator network behaviours explored in phase-isostable dynamics

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Abstract

Following on from the groundwork set in the talk by R Nicks, where it was shown that a phase-isostable reduction captures the behaviour of phase-locked network states with greater accuracy than a higher-order phase reduction, we use that framework to investigate globally coupled neuronal networks of varying size. We consider networks of Morris-Lecar neuron models revealing qualitative correspondence between results from numerical simulations of the full system and the phase-isostable description. We also analyse a network of two coupled next generation neural mass models where phase-isostable reduction offers a dimensional reduction and can capture bifurcations of phase-locked states and emergence of quasiperiodic behaviour which cannot be revealed with a first-order phase description.

The Importance of Model Selection When Studying Gap Junctions

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Abstract

It is well-established that gap junctions play an important role in the synchronisation of neuronal populations, contributing to the generation of both normal and pathological brain rhythms. Unlike coupling through chemical synapses, gap junctions enable ion and molecule exchange between cells in the absence of an action potential. As such, the pre-threshold dynamics, as well as the shape of the action potential, affect the dynamics of gap junction coupled cells.

In this work, we study the dynamics of gap junction coupled Hodgkin-Huxley neurons, leaky integrate-and-fire neurons, Morris Lecar neurons and Izhikevich neurons. We compare the efficiency of the gap junction coupling based on experiments by Galarreta and Hestrin [1],. We highlight discrepancies between the different models and search for an explanation of the discrepancies through studying the model dynamics. We also examine the network-level activity, focusing on the level of synchrony. We study how the gap junction coupling strength affects the level of synchrony, and compare this effect across models. In particular, we quantify the impact of neglecting the shape of the action potential for the LIF model.

References

[1] Galarreta, M., Hestrin, S.: Electrical synapses between Gaba-Releasing interneurons. *Nature Reviews Neuroscience* **2** (2001) 425–433.

Understanding the effect of white matter delays on large scale brain dynamics

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Abstract

The presence of myelin is a powerful structural factor that controls the conduction velocity of mammalian axons. It is the combination of local synaptic activity and non-local delayed axonal interactions within the cortex that is believed to be the major source of large-scale brain signals that are seen in EEG/MEG brain recordings. Here, we present perspectives from neural mass and network modelling and develop a new set of mathematical tools able to unravel the contributions of space-dependent axonal delays to large-scale spatiotemporal patterning of brain activity.

We first analyse a single neuronal population Wilson-Cowan neural mass model with self-feedback and a fixed delay and show how to construct periodic orbits for a Heaviside firing rate. For this nonsmooth model we perform linear stability analysis by augmenting Floquet theory with saltation operations. Building on this example, we then show how to treat the synchronous oscillatory state in networks of nonsmooth neural masses with multiple and heterogeneous delays. To complement this advance in the understanding of synchronous network states and their destabilisation to more novel functional connectivity patterns, we also present numerical simulations (developed in Julia) for both Heaviside and sigmoidal firing rate functions. We use this to highlight the predictive power of the mathematical approach, as well as uncover possible routes to chaos beyond bifurcation.

Finally, we discuss outstanding challenges for when the delays are plastic and state dependent, and present preliminary results for a new form of biologically motivated white matter plasticity rule.

Flux control: Local rules for regulating the working regime of actively processing networks

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Abstract

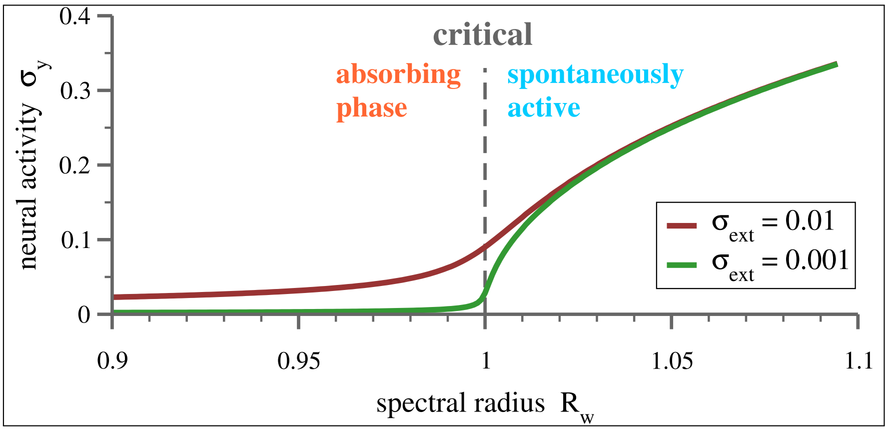
The working regime of a recurrent network is determined by the spectral radius R_w of the synaptic weight matrix. The activity dies out (explodes), modulo non-linear effects, when $R_w < 1$ ($R_w > 1$). The spectral radius, which is given by the largest eigenvalue of the synaptic weight matrix (spectral norm), is a global property. We show that neurons can regulate R_w using only local information by controlling the flow of activity patterns through each neuron, i.e., the ratio between input and output signal variance. The resulting adaptation rule is shown to be robust, leading to a recurrent network configuration that is highly performant when used as an echo state network. Furthermore, the adaption mechanism leads to an inverse square root scaling relation between the synaptic weights and the number of afferent synapses, which is in agreement with experimental data.

Our previous work [1] dealt with the case of uncorrelated random matrices, leading to circular distributions of the eigenvalues in the complex plane. We show that the identical local rule also works when synaptic weights are mostly uni-directional, the typical case in the brain. In this case, the resulting eigenvalue spectrum forms an elliptical distribution in the complex plane.

Furthermore, we develop an analytic theory for the transition between the absorbing ($R_w < 1$) and the active ($R_w > 1$) phase. The variance of the neural activity, σ_y^2, is implicitly determined via

$$2R_w^2(1-\sigma_y^2)^2\sigma_y^2 \;=\; 1-(1-\sigma_y^2)^2(1+2\sigma_{\rm ext}^2) \tag{1}$$

by the spectral radius R_w and the variance of the input, $\sigma_{\rm ext}^2$.



r0.7

This expression becomes exact when neural activites are not correlated. It represents, to our knowledge, the first analytic treatment of an absorbing phase transition in the presence of external drivings.

References

[1] Schubert, Fabian, and Claudius Gros.: Local homeostatic regulation of the spectral radius of echo-state networks. *Frontiers in Computational Neuroscience* **15** (2021) 587721.

Control of neuronal models with applications to Communication Through Coherence

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Abstract

Oscillatory behaviors have been widely observed in the brain across various scales, ranging from single neurons to populations. Notably, oscillations are present in several cognitive tasks, such as perception and attention, and have been hypothesized to play a role in brain communication. The Communication Through Coherence (CTC) theory (Fries, 2005, 2015) provides an explanation for this function. According to CTC, effective communication between populations of neurons requires phase-locking at the appropriate phase of oscillation. In this talk, we aim to explore this phenomenon in excitatory-inhibitory populations of neurons modeled by an exact mean-field model. Our goal is to synchronize populations using control techniques, where the control itself is the action of a higher cortical area (mimicking a top-down mechanism). For this purpose, we will perturb two desynchronized "pre-synaptic" oscillatory populations (sender) to influence a "post-synaptic" population (receiver), which we want to synchronize with one of the two pre-synaptic populations. We will apply a control to the receiving population to enhance synchronization and control the period of the oscillations. To accomplish this, we will use the phase-amplitude reduction of a limit cycle along with optimal-control techniques.

Computing stochastic Phase Response Curves

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Abstract

A theoretical framework allowing to study synchronicity, phase reduction is a powerful mathematical approach to oscillations in which the state of the system is encoded in a single phase variable θ. In particular, this framework allows to compute the Phase Response Curve (PRC), a tool that predicts the response of an oscillatory system to external perturbations, and is used to understand the synchronisation properties of single neurons [1].

However, this approach has limits when it comes to dealing with systems in which fluctuations play a role in defining the oscillatory behaviour of the system, as is often the case in neuroscience. For example, when stimulating a single neuron with a subthreshold input, there exists an optimal level of input noise for which the neuron will exhibit maximally coherent oscillatory spiking: this phenomenon is known as coherence resonance, and is not captured by the deterministic phase reduction approach.

Building on previous work that uses the eigenfunctions of the Kolmogorov operator to characterize stochastic oscillators [2], we build a purely stochastic phase reduction of stochastic oscillators and use it to compute the corresponding *stochastic phase response curve* (sPRC). We show that this approach holds even in the case where the system's oscillations are noise-induced. This is a first step towards an analysis of the synchronisation properties of excitable systems.

References

[1] Winfree, Arthur T. The Geometry of Biological Time. Berlin, Heidelberg: Springer Berlin Heidelberg, 1980.

[2] A. Pérez-Cervera, B. Lindner, and P. J. Thomas, Isostables for Stochastic Oscillators, Phys. Rev. Lett. 127, 254101 (2021).

Fast-slow decomposition in biological excitable systems

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Abstract

Some of the most interesting biological systems, such as neurons and cardiac cells, show excitable behavior. The mathematical models that describe these systems are characterized by combining variables with fast and slow timescales. Singular perturbation theory provides a framework in which these models can be analyzed, highlighting the dynamical properties derived from the differences in timescales [1].

In this talk, the main tools of singular perturbation theory will be presented and applied to the analysis of some of these models. Systems with more than two variables generally present several different timescales, therefore variables can be classified as fast or slow in different ways. The possible fast-slow decompositions of these models will be analyzed, showing how different and complementary information can be obtained about the dynamics, thus proving that a global description of the dynamical properties of excitable systems benefits from this analysis [2].

References

[1] Desroches, M., Guckenheimer, J., Krauskop, B., Kuehn, C., Osinga, H.M., Wechselberger, M.: Mixed-mode oscillations with multiple time scales. *SIAM Review* **54** (2012) 211–288.

[2] Barrio, R., Jover-Galtier, J.A., Martínez, M.A., Pérez, L., Serrano, S.: Mathematical Birth of Early Afterdepolarizations in a Cardiomyocyte Model. *Mathematical Bioscience* (2023). Pre-print.

Numerical continuation approaches for studying the phase response of higher-dimensional oscillators

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Abstract

Phase resetting is a fundamental mechanism in many biological processes, which concerns the shift in the phase of an oscillator in response to an external stimulus. Mathematically, such a response can be studied via a phase transition curve (PTC), which graphs the phase of a point perturbed away from an attracting periodic orbit, as a function of the phase the (fixed-amplitude) perturbation is applied. An isochron is a complementary concept, which is defined as all initial points in space whose trajectories converge to the periodic orbit in phase, and these points form an $n-1$-dimensional sub-manifold. In this talk, we present our method via the continuation of a multi-segment two-point boundary-value problem, to evaluate the consequent phase of perturbed points that converge to the periodic orbit. The set of perturbed points forms a surface parametrized by perturbation amplitude and application phase. The method was originally developed to compute phase transition curves [1], but we also show how it can be extended to compute directly one-dimensional portions of isochron that intersect the surface of perturbed points. We illustrate these methods with 4- and 7-dimensional model examples, which previously encountered numerical difficulties. We show how the methods allow for either instantaneous or time-varying perturbations, and how the isochrons provide a detailed overview of the response of perturbations for models of arbitrary dimension.

References

[1] Langfield, P., Krauskopf, B. and Osinga, H.M.: A continuation approach to computing phase resetting curves. In Advances in Dynamics, Optimization and Computation, Springer, 2020, pp. 3-30.

The intrinsic structure of high-dimensional time series: How to reduce the sample size

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Abstract

Modern techniques such as EEG, MEG, or local field potential (LFP) recordings enable massive acquisition of high-dimensional and high-definition (2HD) data. The analysis of 2HD signals is a challenging problem partly due to the curse of dimensionality. It requires an exponential growth of samples for statistically significant assessment of different measures, which frequently contradicts the data stationarity and availability, especially in medicine. Thus, one of the challenges is the reduction of the number of data samples required for the analysis.

The correlation dimension (CD) is one of the data complexity measures. Initially, it was introduced to describe the geometric structure of chaotic attractors, but it has rapidly been extended into biological time series. The CD can categorize EEGs of patients with different brain disorders [1]. However, there is controversy over its usage. A reduced sample size of the available data can lead to inaccurate estimates of the CD. In this talk, we present mathematical results that use independent component analysis adapted to LFPs [2] that enable a significant reduction of the sample size of time series to estimate the CD accurately. Then, we discuss the application of the technique to LFPs collected in the rat hippocampus and comment on its significance for information processing.

References

[1] Maimaiti, B., Meng, H., Lv, Y., et al.: An Overview of EEG-based machine learning methods in seizure prediction and opportunities for neurologists in this field. *Neuroscience* **481** (2022) 197–218.

[2] Makarov, V.A., Makarova, J., Herreras, O.: Disentanglement of local field potential sources by independent component analysis. *J. Comput. Neurosci.* **29** (2010) 445–457.

Modelling and Analysis of Pharmacological-Induced Mixed-Mode Oscillation in a Neuronal Model

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Abstract

A recent neurobiological study presented in [1] recorded neuronal electrical activity classifiable as Mixed-Mode Oscillations (MMOs). This work proposes a 5D Hodgkin-Huxley type model to reproduce the observed phenomena under various experimental conditions, and to explain the underlying dynamical structures. We suggest that MMOs are driven via modulation of HCN and M channels by cAMP. To support this thesis the ODE system is first reduced to three dimensions where the very slow activation variables of both HCN (r) and M (w) channels are considered as parameters. Thanks to this simplification, one- and two-dimensional bifurcation diagrams with bifurcation parameters r and w are derived. MMO evolution arises when the pair (w, r) belongs to a region limited by Hopf and Period Doubling bifurcations. The small-amplitude oscillations are generated via Folded Node (FN), respectively Singular Hopf Bifurcation, mechanisms, when the unstable equilibrium and the FN are sufficiently distant, respectively close. To understand how the local flow is organized, canard orbits, repelling and attracting slow manifold are reconstructed numerically, and the results are interpreted via Geometrical Singular Perturbation Theory (GSPT). Bifurcation diagrams of the 3D model are then used to explain the 5D temporal evolution leading to the conclusion that the observed MMO activity is controlled by both HCN and M channels.

References

[1] Lezmy, J., Arancibia-Cárcamo, I.L., Quintela-López, T., Sherman, D.L., Brophy, P.J., Attwell, D.: Astrocyte Ca^{2+}-evoked ATP release regulates myelinated axon excitability and conduction speed. *Science*, **374**, (2021), eabh2858.

Neural coding of subthreshold periodic signals in symbolic spike patterns

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Abstract

The biophysical mechanisms by which external inputs elicit electrical responses from a neuronal ensemble are well known; however, a good understanding of how neurons use these spikes to encode the input information is still lacking. Single neurons and neuronal populations use different coding mechanisms depending on the type of input. Sensory neurons encode sensory inputs using a 'sparse coding' strategy, by which only a small number of neurons are simultaneously active at a given time. Moreover, neuronal responses at different timescales can encode different features of an input.
In this presentation, I will discuss the activity of a neuronal ensemble, with individual neurons described by the FitzHugh-Nagumo model, when all neurons perceive a subthreshold sinusoidal input. I will discuss how the probabilities of symbolic ordinal spike patterns [1], computed from the spike sequences of all neurons, depend on the amplitude and period of the input, the noise level, the number of neurons in the ensemble, and the couplings between them (link density, link strength) [2]. I will show that the symbolic probabilities carry information about the input and display resonance-like extreme values with respect to input parameters (amplitude, frequency) and noise intensity. I will also show that just a few connections can significantly improve signal encoding. Our findings suggest that encoding information in infrequent and preferred spike ordinal patterns is a plausible mechanism that neural populations can use to encode weak inputs in noisy environments.

References

[1] Leyva, I., Martinez, J.M., Masoller, C., Rosso, O.A., Zanin, M.: 20 Years of Ordinal Patterns: Perspectives and Challenges. *EPL* **138** (2022) 31001.

[2] Masoliver, M., Masoller, C.: Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns. *Commun. Nonlinear Sci. Numer. Simulat.* **88** (2020) 105023.

Coupling Neural Populations

A. Mayora-Cebollero [1], **R. Barrio** [1], **J.A. Jover-Galtier** [1],
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Abstract

Mean-field models allow us to study the average behaviour of a population of a large number of neurons. If we couple two mean-fields, we can study the dynamics of two coupled populations.

In the literature, the coupling of two mean-field models, each one describing the macroscopic quantities of a network of quadratic integrate-and-fire neurons (with synaptic currents), can be found in [1]. If we set to one the membrane time constant of both populations, the limit case in which the synaptic time constants are zero corresponds to another coupled mean-field model in the literature [2] (that does not involve the synaptic currents, only the strength of the synaptic connections). In this talk, we apply different techniques to study the dynamics when the value of the synaptic time constants varies [3].

References

[1] Dumont, G., Gutkin, B.: Macroscopic phase resetting-curves determine oscillatory coherence and signal transfer in inter-coupled neural circuits. *PLoS computational biology* **15(5)** (2019).

[2] Montbrió, E., Pazó, D., Roxin, A.: Macroscopic description for networks of spiking neurons. *Physical Review X* **5(2)** (2015).

[3] Barrio, R., Jover-Galtier, J.A., Mayora-Cebollero, A., Mayora-Cebollero, C., Serrano, S.: Different dynamic regimes when coupling neural populations. *Preprint* (2023).

GABA-glutamate functional mechanisms in the posterodorsal medial amygdala: in-silico approach

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Abstract

Posterodorsal medial amygdala (MePD) is a limbic brain structure that is implicated in the modulation of sociosexual and reproductive functions in mammals by projecting onto the neuroendocrine areas in the hypothalamus. Neuronal activity within the region is regulated by neuropeptide kisspeptin that provides excitatory input to glutamatergic projection neurons and GABAergic interneurons, which in turn inhibit GABA projections [1]. Despite extensive research into GABA-glutamate dynamics in the MePD, the exact functional mechanism underlying the neuronal interactions within the network is unknown. We propose a coarse-grained model, based on the Wilson-Cowan framework, that captures the interplay of cooperative and competitive dynamics between GABA and glutamate within the MePD. We employed bifurcation analysis to investigate the effects of the connectivity strength between populations and upstream input from kisspeptin on qualitative behaviour in the network. Furthermore, we also demonstrated (in the model) how MePD output modulates neuronal populations in the arcuate nucleus, a small region in the hypothalamus of the brain, that plays an important role in regulating many physiological processes such as appetite, metabolism, and reproduction.

References

[1] Lass, G., Li, X.F., Voliotis, M., Wall, E., de Burgh, R.A., Ivanova, D., McIntyre, C., Lin, X., Colledge, W.H., Lightman, S.L., Tsaneva-Atanasova, K. and O'Byrne, K.T. GnRH pulse generator frequency is modulated by kisspeptin and GABA-glutamate interactions in the posterodorsal medial amygdala in female mice. *Journal of Neuroendocrinology* (2022).

Insights into neural oscillator network dynamics using a phase-isostable framework

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Abstract

Networks of coupled nonlinear oscillators can display a wide range of emergent behaviours under variation of the strength of the coupling. Of particular interest in this talk are neuronal networks where nodes are single neuron or neural population models. In these cases, interactions may be significant in magnitude compared to the rate of decay to the underlying stable limit cycle. Since the standard technique of first-order phase reduction breaks down beyond the weak coupling regime it therefore fails to capture many important features of the dynamics of these neural networks. Recent work has shown isostable coordinates to be a useful concept to characterise the transient behaviour of oscillators in directions where decay to the limit cycle is slow. Reduced network equations for two coupled oscillators, where the dynamics of each node is described by the evolution of its phase and slowest decaying isostable coordinate, have been shown to capture bifurcations and dynamics of the network which cannot be explained through standard phase reduction. An alternative framework using isostable coordinates to obtain higher-order phase reductions has also demonstrated a similar descriptive ability for two oscillators.

In this talk we discuss the extension of phase-isostable network equations to an arbitrary but finite number of coupled oscillators, giving conditions required for stability of phase-locked states including synchrony. For examples where the dynamics of the full system are known, we compare the accuracy of the phase-isostable network equations and higher-order phase reductions in capturing bifurcations of phase-locked states. We find the former to be the more accurate and therefore we may employ this framework to investigate the dynamics of a number of globally coupled neuronal networks of varying size for both planar node models and higher dimensional examples where the phase-isostable description can offer a dimensional reduction. This will be discussed in detail in the talk of Robert Allen.

Coupling dynamical systems with a Hopf bifurcation

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Abstract

We consider the following coupling of two identical Fitzhugh-Nagumo systems:

$$\left\{\begin{array}{rcl} x_1' &=& c(y_1 + x_1 - \frac{x_1^3}{3}) + \alpha_1(x_2 - x_1), \\ y_1' &=& -\frac{1}{c}(x_1 - a + by_1) + \alpha_2(y_2 - y_1), \\ x_2' &=& c(y_2 + x_2 - \frac{x_2^3}{3}) + (\alpha_1 + \varepsilon_1)(x_1 - x_2), \\ y_2' &=& -\frac{1}{c}(x_2 - a + by_2) + (\alpha_2 + \varepsilon_2)(y_1 - y_2), \end{array}\right.$$

where

$$0 < b < 1, \quad c > 0, \quad b < c^2,$$

$\alpha_i \in R$ and $\varepsilon_i \in R$. Initially, our model works with an asymmetric coupling between neuron, that can be made symmetric by the condition $\varepsilon_1 = \varepsilon_2 = 0$. The plane $\Pi = \{(x_1, y_1, x_2, y_2) \,|\, x_1 = x_2,\, y_1 = y_2\}$ is invariant by the flow of the above system and the restricted dynamics are the ones of an isolated Fitzhugh-Nagumo system; therefore, there exists a Hopf bifurcation on Π. Our goal is to study bifurcations that can arise due to additional degeneracies occurring in directions transverse to Π. Namely, we consider the simplest possibilities: codimension-two Hopf-pitchfork and Hopf-Hopf bifurcations. Different cases are detected and classified in the general model and the dynamical consequences are discussed. Similar four-dimensional models have been analyzed in [1] and more recently in [2].

References

[1] S. A. Campbell, M. Waite: Multistability in coupled Fitzhugh-Nagumo oscillators. *Proceedings of the Third World Congress of Nonlinear Analysts* **47** (2001), 1093-1104.

[2] L. Santana, R. M. da Silva, H. A. Albuquerque, C. Manchein: Transient dynamics and multistability in two electrically interacting Fitzhugh-Nagumo neurons. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **31 (5)** (2021) 053107.

Building generalized cognitive maps of 2D dynamic situations by nonlinear 3D waves

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Abstract

A generalized cognitive map (GCM) is a functional mental representation of a dynamic situation that can be used by an individual for path planning when obstacles and/or targets move in the environment [1]. The mechanisms of building GCMs and their fundamental properties have yet to be understood [2]. Here, we discuss a three-dimensional (3D) reaction-diffusion model capable of generating GCMs for 2D environments. One extra dimension of the model allows the embedding of a great variety of velocity-changing 2D trajectories in a single static GCM. We develop an efficient numerical scheme for integrating the model and illustrate it using examples of typical dynamic situations. The found biophysical mechanisms can endow robots with a human-like basic cognition.

References

[1] Villacorta-Atienza, J.A., Makarov, V.A.: Neural network architecture for cognitive navigation in dynamic environments. *IEEE Transactions on Neural Networks and Learning Systems* **24** (2013) 2075–2087.

[2] Villacorta-Atienza, J.A., Calvo Tapia, C., Diez-Hermano, S., Sanchez-Jimenez, A., Lobov, S., Krilova, N., Murciano, A., Lopez-Tolsa, G., Pellon, R., Makarov V.A.: Static internal representation of dynamic situations reveals time compaction in human cognition. *Journal of Advanced Research* **28** (2021) 111–125.

Dissecting the bifurcation skeleton of the spike-adding phenomena

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Abstract

Bursting is a common and relevant phenomenon regarding neuron dynamics. In this talk our focus will be the different spike-adding processes than can take place among models exhibiting fold/homoclinic and fold/Hopf bursting. We identify three different processes in different neuron models: the canard-induced case, the chaos-induced case and the Hopf-induced case. Separately, these cases were shown in the literature, but our perspective allows to build a spike-adding map that explains their location with respect to the homoclinic bifurcation structure, and explains how the transitions between different spike-adding processes take place. We consider the Hindmarsh-Rose model, a prototypical model of fold/homoclinic and fold/Hopf bursting, and the Sherman model of the pancreatic betacell, a more realistic, biophysiologically detailed model.

References

[1] R. Barrio, S. Ibáñez, L. Pérez, S. Serrano: Classification of fold/hom and fold/Hopf spike-adding phenomena. *Chaos* **(4)**:043120 (2021).

[2] R. Barrio, S. Ibáñez, L. Pérez, S. Serrano. Spike-adding maps in bursting models. *Work in progress.*

A Universal Description of Stochastic Oscillators

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Abstract

Many systems in physics, chemistry and biology exhibit oscillations with a pronounced random component. Such stochastic oscillations can emerge via different mechanisms, for example linear dynamics of a stable focus with fluctuations, limit-cycle systems perturbed by noise, or excitable systems in which random inputs lead to a train of pulses. Despite their diverse origins, the phenomenology of random oscillations can be strikingly similar. Here we introduce a nonlinear transformation of stochastic oscillators to a new complex-valued function $Q_1^*(\mathbf{x})$ that greatly simplifies and unifies the mathematical description of the oscillator's spontaneous activity, its response to an external time-dependent perturbation, and the correlation statistics of different oscillators that are weakly coupled. The function $Q_1^*(\mathbf{x})$ is the eigenfunction of the Kolmogorov backward operator with the least negative (but non-vanishing) eigenvalue $\lambda_1 = \mu_1 + i\omega_1$. The resulting power spectrum of the complex-valued function is exactly given by a Lorentz spectrum with peak frequency ω_1 and half-width μ_1; its susceptibility with respect to a weak external forcing is given by a simple one-pole filter, centered around ω_1; and the cross-spectrum between two coupled oscillators can be easily expressed by a combination of the spontaneous power spectra of the uncoupled systems and their susceptibilities. Our approach makes qualitatively different stochastic oscillators comparable, provides simple characteristics for the coherence of the random oscillation, and gives a framework for the description of weakly coupled oscillators.

Pulse shape and voltage-dependent synchronization in spiking neuron networks

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Abstract

Pulse-coupled spiking neural networks are a powerful tool to gain mechanistic insights into how neurons self-organize to produce coherent collective behavior. These networks use simple spiking neuron models, such as the θ-neuron or the quadratic integrate-and-fire (QIF) neuron, that replicate the essential features of real neural dynamics. Interactions between neurons are modeled with infinitely narrow pulses, or spikes, rather than the more complex dynamics of real synapses. To make these networks biologically more plausible, researchers have proposed that they must also account for the finite width of the pulses, which can have a significant impact on the network dynamics. However, the derivation and interpretation of these pulses is contradictory and the impact of the pulse shape on the network dynamics is largely unexplored. Here, I take a comprehensive approach to pulse-coupling in networks of QIF and θ-neurons. I argue that narrow pulses activate voltage-dependent synaptic conductances and show how to implement them in QIF neurons such that their effect can last through the phase after the spike. Using an exact low-dimensional description for networks of globally coupled spiking neurons, I prove for instantaneous interactions that collective oscillations emerge due to an effective coupling through the mean voltage. I analyze the impact of the pulse shape by means of a family of smooth pulse functions with arbitrary finite width and symmetric or asymmetric shapes. For symmetric pulses, the resulting voltage-coupling is little effective in synchronizing neurons, but pulses that are slightly skewed to the phase after the spike readily generate collective oscillations. The results unveil a voltage-dependent spike synchronization mechanism, facilitated by pulses of finite width and complementary to traditional synaptic transmission, at the heart of emergent collective behavior of spiking neurons.

Rethinking selection criteria for data-driven models

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Abstract

As we seek to understand more complex neural phenomena, so too must our models become more complex. Increasingly such models can be fitted directly in a data-driven manner [1]. However because the fitting problem is typically non-convex, this yields many possible parameters wich cannot be differentiated using established model comparison methods.
Our proposed solution begins with an ontological argument: we argue that it is essential to distinguish the *physical model* – which is grounded in theory – from the *observation model*. The parameters of the latter are less important, since its role is just to account for the mismatch between theory and data. In fact, we propose to forego parameterising the observation model altogether to allow for "unknown unknowns". Instead, we compare parameter sets based on an average over the space of all possible observation models.
We then proceed to develop a practical method for computing the expectation over unparameterised observation models. This is built upon three main ideas. The first is to cast expectations as path integrals in a space of 1d quantile functions. The second is then a method for sampling quantile paths, and thus estimating the path integral numerically. The third is a calibration procedure, which allows practitioners to adjust the criterion's sensitivity to the specifics of the model and the data. Finally we apply the method to compare fitted Hodgkin-Huxley and Wilson-Cowan models.

References

[1] René, A., Longtin, A., Macke, J. H.: Inference of a Mesoscopic Population Model from Population Spike Trains. *Neural Comput.* **32** (2020) 1448–1498

Macroscopic description of spiking neural networks with spike frequency adaptation and synaptic plasticity

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Abstract

Neuronal networks are endowed with a multitude of mechanisms that modulate their spiking activity and connectivity structure, such as spike frequency adaptation (SFA) and short-term synaptic plasticity. Typically, these mechanisms add long time scales on top of membrane time scales and synaptic transmission times, and are commonly studied using large-scale simulations at the spiking neuron level. We present methods for deriving the macroscopic equations for networks composed of non-identical quadratic integrate-and-fire neurons (based on [1]) that incorporate such mechanisms.

For the first part, we focus on SFA and analyze the resulting equations by means of bifurcation analysis. SFA leads to the emergence of bursting behavior, which can coexist with steady state behavior providing a bistable regime that enables transient switches between synchronized and non-synchronized states of population dynamics. The macroscopic equations provide an accurate and efficient description of phase transitions between bursting and steady state population dynamics, which are organized by Hopf and homoclinic bifurcations. We also show that the macroscopic equations generalize well to networks of biologically plausible size and coupling probability [2].

In the second part, we incorporate short-term synaptic depression and facilitation into the model equations. The straightforward derivation of the macroscopic equations is no longer possible in this case, and we propose two approaches to obtain an approximate macroscopic description: (a) an adiabatic approximation of the network dynamics, which yields a low-dimensional, but somewhat inaccurate description of the macroscopic dynamics; and (b) a more accurate multi-population approximation of the

network dynamics. The latter is based on dividing the network into subnetworks of neurons with nearly identical properties, for which the macroscopic dynamics can be easily obtained. Both approaches outperform standard mean field methods in terms of their accuracy [3].

Our work provides a comprehensive account of the effects of SFA and short-term synaptic plasticity on the macroscopic dynamics of spiking neural populations, and the mathematical tools to predict population-level effects of SFA and short-term plasticity from single neuron properties.

References

[1] Montbrió, E., Pazó, D., Roxin, A.: Macroscopic description for networks of spiking neurons. *Phys. Rev. X* **5** (2015) 021028.

[2] Gast, R., Schmidt, H., Knösche, T. R.: A mean-field description of bursting dynamics in spiking neural networks with short-term adaptation. *Neur. Comp.* **32** (2020) 1615-1634.

[3] Gast, R., Knösche, T. R., Schmidt, H.: Mean-field approximations of networks of spiking neurons with short-term synaptic plasticity. *Phys. Rev. E* **104** (2021) 044310.

Modeling of dynamical systems: Data-driven Koopman operators and quantum mechanics

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Abstract

A framework for data assimilation and prediction of nonlinear dynamics is presented, combining aspects of quantum mechanics, Koopman operator theory, and kernel methods for machine learning. This approach adapts the formalism of quantum dynamics and measurement to perform data assimilation (filtering), using the Koopman operator governing the evolution of observables as an analog of the Heisenberg operator in quantum mechanics, and a quantum mechanical density operator to represent the data assimilation state. The framework is implemented in a fully empirical, data-driven manner by representing the evolution and measurement operators via matrices in a basis learned from time-ordered observations. Applications to data assimilation of the Lorenz 96 multiscale system and others show promising results. Extensions of this work toward spatiotemporal pattern extraction and subgrid-scale modeling is presented as well, with the particular emphasis on its suitability for computational neuroscience research. Furthermore, our framework provides a route for implementing and designing appropriate algorithms on quantum computers.

References

[1] N. Marrouch, J. Slawinska, D. Giannakis, and H. Read, 2020: Data-driven Koopman operator approach for computational neuroscience. Annals of Mathematics and Artificial Intelligence volume. doi: 10.1007/s10472-019-09666-2.

[2] J.Slawinska, D. Giannakis, and A. Ourmazd, 2019: A quantum mechanical approach for data assimilation in climate dynamics. International Conference on Machine Learning Workshop on "Climate Change: How Can AI Help?".

[3] D. Freeman, D. Giannakis, B. Mintz, A. Ourmazd and J. Slawinska, 2023: Data Assimilation in Operator Algebras. Proceedings of the National Academy of Sciences, doi:10.1073/pnas.2211115120.

Slow passage through the transcritical bifurcation by PWL systems

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Abstract

Given a uniparametric differential equation that undergoes a bifurcation at one value of the parameter, one can consider the effect on the entire flow when introducing slow dynamics to the parameter and making it varying around the bifurcation value. This procedure drives to a fast slow system. Some effects on the global flow that appear through this dynamic bifurcation are easily predictable in the context of Fenichel's theory, but other effects are more surprising and we call them slow passage phenomena through the bifurcation. One of these surprising effects associated with the loss of stability of an equilibrium is the appearance of a non-trivial delay in the loss of stability. The occurrence of this delay is a key element in the analysis of elliptic bursting oscillations.

In this session we will use the piecewise linear context to describe quantitatively and qualitatively the nature of the occurrence of such a delay in the loss of stability when passing through a transcritical bifurcation. We will also apply the obtained results to the analysis of an excitable PWL model exhibiting a canard explosion via two transcritical bifurcations.

Network topology and patterns in small CPGs

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Abstract

We present some of the past and recent results we have obtained on global synchronization of small networks of neurons. In previous papers [2, 3] we proposed some numerical techniques to detect patterns in Central Pattern Generators (CPGs) and we applied these techniques to a 6-neuron CPG modeling insect movement (based on Ghigliazza and Holmes [1] model for motoneurons in cockroaches). Tripod gait appeared as the dominant pattern, as it does in Nature, and this dominance seemed to be quite robust under parameter changes of the network. This suggested to us that the dominance of the tripod gait could be a consequence of network's topology.

Tripod gait has a natural generalization to CPGs whose underlying graph is, as in Ghigliazza and Holmes model, a bipartite graph, and we wonder if these "bipartite patterns" are also dominant among the syncronization patterns of bipartite CPGs. We have extended our previous study to all bipartite CPGs with at most 9 neurons. We will present the results of this new study with other topological considerations that arise in this context.

References

[1] Ghigliazza, R.M., Holmes, P.: A Minimal Model of a Central Pattern Generator and Motoneurons for Insect Locomotion, *SIAM Journal on Applied Dynamical Systems*, **3(4)** (2004) 671–700.

[2] Barrio, R., Lozano, Á., Rodríguez, M. and Serrano, S. Numerical detection of patterns in CPGs: Gait patterns in insect movement, *Communications in Nonlinear Science and Numerical Simulation*, **82** (2020) 105047.

[3] Barrio, R., Lozano, Á., Martínez, M.A., Rodríguez, M., Serrano S.: Routes to tripod gait movement in hexapods, *Neurocomputing* **461** (2021) 679–695.

Oscillations in a piecewise linear rate model with firing adaptation

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Abstract

A deterministic variant of a population based firing rate model is studied which was built by D. Jercog et al. [1]. The model describes the dynamical behaviour of an excitatory and an inhibitory population which are recurrently connected. Adaptation current is also added to the system as a negative feedback of excitatory cells. Shifted rectified linear unit function is applied as activation function which gives the nonlinearity of the system.

Our goal is to study the appearance of different types of oscillations which can be observed in neurobiological experiments. Bifurcation theory is used to find those parameter values where the solutions can change significantly. We focus on the effect of adaptation and the self-weight of the excitatory population. The number of equilibria and also their stability are determined. Hopf bifurcation is studied which results in birth of periodic orbits. The applied piecewise linear activation function allows us to give explicit formulas for the curves of local bifurcations. Some examples for the most interesting phase portraits are shown including bistability and periodic solutions. Unstable limit cycle is detected in the three-dimensional phase space via Poincaré map and its coexistence with stable periodic orbit is also shown.

References

[1] Jercog, D., et al.: UP-DOWN cortical dynamics reflect state transitions in a bistable network. *Elife* **6** (2017): e22425.

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6 OTHER INFORMATION

All the talks (and reception) will take place at the CIEM center, 4 Maria Aburto street.

Lunches

Lunches will be served at Hotel Las Rocas (1 Flaviobriga street) at 13:30.

Figure 1: Location of the CIEM center and Hotel Las Rocas.

Social Dinner

The Social Dinner will be on Thursday 15th at 21:00 at Hotel Las Rocas (1 Flaviobriga street).